MODEL QUESTION FOR END SEMESTER

B.Sc. Sem – VI Paper – XVI

Numerical Analysis + Metric Space

Short type question:

1) Define Forward Difference operator, Backword Difference operator and central Difference operator.

2) Prove that
   a) The operators Δ and E are linear.
   b) The operators Δ and E are commutative
   c) $\Delta[f(x)g(x)] = \{Ef(x)\}\cdot\Delta g(x) + g(x)\cdot\Delta f(x)$.

   d) $\Delta x^{(n)} = n x^{(n-1)}$

   e) $\nabla = \Delta E^{-1} = E^{-1}\Delta = I - E^{-1}$.

3) If

x:	1	2	3	4	5
y:	2	5	10	20	30

find the forward difference table $\Delta^4 y(1)$.

4) obtain the estimate of the missing figure in the following table

x:	1	2	3	4	5	6	7	8
$f(x)$:	1	8	?	64	?	216	343	512

5) Evaluate by Newton-Raphson method
   i) $\sqrt{29}$

6) Define an open sphere.
Prove that a subset G of a metric space (X,d) is open iff G is a union of open spheres.

7) Let (X,d) be a metric space and F be any subset of X. Prove that F is closed iff $F=\bar{F}$

8) Let R be the set of all real nos. and $d: R\times R \longrightarrow R$ defined by $d(x,y) = |x-y| \;\forall\; x,y \in R$. then Prove that d is a metric on R.

9) Define
i) Neighbourhood of a point
ii) Interior point of a set.
iii) Limit point of a set
iv) closure of a set
v) Cauchy sequence and complete Metric Space

10) Give an example which shows that the intersection of an arbitrary collection of open set is not open.

Long-Type questions:

1) State and Prove Fundamental Theorem of difference Calculus.

2) Show that

$$x^{(-n)} = \frac{1}{(x+n)^{(n)}}$$

the interval of differencing being one.

3) Find a real root of the equation $x^3 - x - 1 = 0$ By Bisection Method.

4) Evaluate $\int_0^6 \frac{dx}{1+x^2}$

By using i) Simpson's '1/3' rule
ii) Simpson's '3/8' rule

5) Calculate by Simpson's rule an approximate value of $\int_{-3}^{3} x^4 dx$ by taking seven equidistant ordinates. Compare it with the exact value and the value obtained by using the Trapezoidal rule.

6) Let R^n be the set of all n-tuples of real numbers given by $x = (x_1, x_2, \dots x_n)$ $\forall x_i \in R$ $i = 1, 2, 3, \dots n$
We define a mapping $d_2 : R^n \times R^n \to R$ given by

$$d_2(x, y) = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}$$

Then show that (R^n, d_2) is a metric space.

7) Let (X, d) be a metric space then prove that each pair x, y of distinct points of X, $\exists$ a neighbourhood U of x and a neighbourhood V of y such that $U \cap V = \phi$

8) State and Prove Cantor's Intersection Theorem.

9) Show that in a metric space (X, d) Every convergent sequence is Cauchy sequence. Also show that the converse is not always true.

10) Let (X, d) be any metric space and F is any subset of X then F is closed iff it contains each of its limit point. Prove it